

Scheduling model of manufacturing systems using tropical algebra

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Abstract:

The production process is included in the Discrete Event System (DES). The DES independent variable generally depends on the event, so an event is influenced by the previous event. Tropical algebra can be used in the DES system to change the nonlinear system of equations to linear equations. Whereas Tropical Algebra was introduced after the 1980s, the number of studies regarding this tool and its applications is fewer than regarding Discrete Even Simulation and Queuing models. Consequently, tropical Algebra needs to be applied and tested in many systems in order to explore hidden aspects of its function and capabilities. The purpose of this study is to provide a scheduling model using tropical algebra, for a specific class of discrete event system which are specifically manufacturing systems. Such systems are called tropical linear discrete event systems. The methods used is constructing matrix equations from the system of equations, solving Eigen value and Eigen vectors to determine the schedule production and find starting time. The results of the research showed that the production system run periodically every 7 hours. Also the best time for each unit to start working was determined, as well as the production schedule base on daily working hours is obtained.

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1. Introduction

Recently, both industry and academia are evolving interest in techniques to model, analyze, and control complex discrete-event systems (DESs) such as flexible manufacturing systems, although in general, DESs lead to a nonlinear description in conventional algebra, there exists a subclass of DESs for which its model becomes "linear" when it is formulated in tropical algebra Seleim [1]. Silva [2] described DES as a subject in the discipline of systems theory and control which includes artificial systems with a finite amount of resources. The DES independent variable generally depends on the event, not on time, such that an event is influenced by a previous event, not by the current time. The concept of Eigen value and Eigen vectors over tropical algebra, which allows for a unified treatment of many problems that arise in the field of operations research will be used to illustrate or determine the schedule production/manufacturing planning.

The larger the demand for a specific product in the particular area increased. The more production system management needs to be supported by improving production. Determination of an efficient production schedule is one of the requirements for good management so that this problem then becomes the basic idea of the study. However the objective of this research is to develop scheduling model of a production system and determines the starting time for each task in the scheduled production. Sulaiman [3].

There have been a few scholars who applied tropical algebra in flow-line discrete manufacturing systems including Majdzik, *et al.* [4], Vazquez and Alexei [5], Krivulin [6]. However, there are other types of manufacturing systems processes and configurations, such as repetitive, continuous, rail networks, and

job shop, which are modeled and tested using this mathematical tool. Komenda, *et al.* [7] described how tropical algebra became an important tool in discrete event systems models. Al Bermanei *et al.* [8], conduct a research on modeling and scheduling of production system using tropical algebra.

Most of previous studies like De Schuter [9], Selem and Elmaraghy [10], Jaroslow [11] and Imaev [12] use a less number of processors and the type of production system used are serial. Als Ali [13], Li [14] and Imaev and Judd [15] used combination of parallel stations and buffers. In contrast to previous studies, in this study more processors were used, 6 stages that uses an assembly type production system with single input in it so that it better describe the real problem in the field.

2. Development of scheduling model

2.1 Assumptions of the model.

The assumption of the study are as follows;

- Process time of production Stage(s) P_i is deterministic and fixed.
- The temporary storages between two parallel stages (P_2 and P_3) and (P_4 and P_5) respectively are only limited by the storage time, not by their size.
- All production Stage(s) P_i have infinite buffers.
- The process sequence is based on production system flow line.
- The production Stage(s) P_i can only processes one part at a time.

2.2 Notations

The following notations and symbols were used in the development of the model

k : is an event counter, which is the number of parts (jobs),
 d_i : Denotes processing time of production Stage(s) P_i ,
 t_i : Denotes transportation time of production Stage(s) P_i ,
 $u(k)$: Denotes the time instants at which incoming part k is fed to the production Stage(s) P_i ,
 $x_i(k)$: Denotes the time instants when production Stage(s) P_i starts processing part k ,

$Y(k)$: Stands for the time instants at which part k is completed and leaves the line as a finished product.
 P_i : Denoted the production stage.
 S_1 and S_2 : Denoted Temporary Storage one and two respectively.

3. The scheduling model

Using the assumption stated and the notations the graphical representation of the model is shown in Fig. 1.

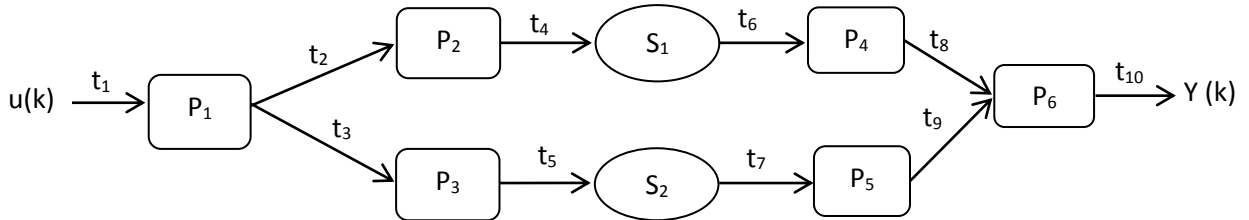


Figure 1: Production system diagram

3.1 Derivation of mathematical model

First we determine $x_1(k)$, i.e, the time instant at which P_1 start working for the k^{th} time. If we feed raw material to the system for the k^{th} time, then this raw material is available at the input of processing unit P_1 at time $t = u(k) + t_1$. However P_1 can only start working on the new batch of raw material as soon it has finish processing the current, i.e. the $(k-1)^{th}$ intermediate product will be P_1 at time $t = x_1(k-1) + d_1$. Since P_1 start working on a batch of raw material as soon as raw material is available and the current batch has left the processing unit, this implies that we have

$$x_1(k) = \max(x_1(k-1) + d_1, u(k) + t_1) \quad (1)$$

Using similar reasoning, we obtain the following difference equations;

$$x_2(k) = \max(x_1(k) + d_1 + t_2, x_2(k-1) + d_2) \quad (2)$$

$$x_3(k) = \max(x_1(k) + d_1 + t_3, x_3(k-1) + d_3) \quad (3)$$

$$x_4(k) = \max(x_2(k) + d_2 + t_4) \quad (4)$$

$$x_5(k) = \max(x_3(k) + d_3 + t_5) \quad (5)$$

$$x_6(k) = \max(x_4(k) + t_6, x_4(k-1) + d_4) \quad (6)$$

$$x_7(k) = \max(x_5(k) + t_7, x_5(k-1) + d_5) \quad (7)$$

$$x_8(k) = \max(x_6(k) + t_8, x_7(k) + t_9, x_6(k-1) + d_6) \quad (8)$$

$$y(k) = x_8 + d_6 + t_{10} \quad (9)$$

In order to obtain an equation as a tropical linear of the form

$$x(k) = A \otimes x(k-1) \oplus B \otimes U(k).$$

Right hand sides of equation (1) – (9) by some straight forward algebraic manipulations and simplifications become;

$$x_1(k) = \max(x_1(k-1) + d_1, u(k) + t_1) \quad (10)$$

$$x_2(k) = \max(x_1(k-1) + 2d_1 + t_2, x_2(k-1) + d_2, u(k) + t_1 + d_1 + t_2) \quad (11)$$

$$x_3(k) = \max(x_1(k-1) + 2d_1 + t_3, x_3(k-1) + d_3, u(k) + t_1 + d_1 + t_3) \quad (12)$$

$$x_4(k) = \max(x_1(k-1) + 2d_1 + t_2 + d_2 + t_4, x_2(k-1) + 2d_2 + t_4, u(k) + \max(t_1 + d_1 + t_2 + d_2 + t_4, d_2 + t_4)) \quad (13)$$

$$x_5(k) = \max(x_1(k-1) + 2d_1 + t_3 + d_3 + t_5, x_3(k-1) + 2d_3 + t_5, u(k) + \max(t_1 + d_1 + t_3 + d_3 + t_5, d_3 + t_5)) \quad (14)$$

$$x_6(k) = \max(x_1(k-1) + 2d_1 + t_2 + d_2 + t_4 + t_6, x_2(k-1) + 2d_2 + t_4 + t_6, x_4(k-1) + d_4, u(k) + \max(t_1 + d_1 + t_2 + d_2 + t_4 + t_6, d_2 + t_4 + t_6)) \quad (15)$$

$$x_7(k) = \max(x_1(k-1) + 2d_1 + t_3 + d_3 + t_5 + t_7, x_3(k-1) + 2d_3 + t_5 + t_7, x_5(k-1) + d_5, u(k) + \max(t_1 + d_1 + t_3 + d_3 + t_5 + t_7, d_3 + t_5 + t_7)) \quad (16)$$

$$x_8(k) = \max(x_1(k-1) + 2d_1 + t_8 + \max(t_2 + d_2 + t_4 + t_6, t_3 + d_3 + t_5 + t_7), x_2(k-1) + 2d_2 + t_4 + t_6 + t_8, x_3(k-1) + 2d_3 + t_5 + t_7 + t_8, x_4(k-1) + d_4 + t_8, x_5(k-1) + d_5 + t_8, x_6(k-1) + d_6, u(k) + \max(t_1 + d_1 + t_2 + d_2 + t_4 + t_6 + t_8, d_1 + t_2 + t_3 + d_3 + t_5 + t_7 + t_8)) \quad (17)$$

Letting

$$\delta_1 = d_1 + t_1 + t_2$$

$$\delta_2 = d_1 + t_1 + t_3$$

$$\delta_3 = \max(d_1 + t_1 + t_2 + d_2 + t_4, d_2 + t_4) = d_1 + t_1 + t_2 + d_2 + t_4$$

$$\delta_4 = \max(d_1 + t_1 + t_3 + d_3 + t_5, d_3 + t_5) = d_1 + t_1 + t_3 + d_3 + t_5$$

$$\delta_5 = \max(d_1 + t_1 + t_2 + d_2 + t_4 + t_6, d_2 + t_4 + t_6) = d_1 + t_1 + t_2 + d_2 + t_4 + t_6$$

$$\delta_6 = \max(d_1 + t_1 + t_3 + d_3 + t_5 + t_7, d_3 + t_5 + t_7) = d_1 + t_1 + t_3 + d_3 + t_5 + t_7$$

$$\delta_7 = \max(d_2 + t_2 + t_4 + t_6, d_3 + t_3 + t_5 + t_7) = d_1 + t_1 + t_2 + d_2 + t_4$$

$$\delta_8 = \max(d_1 + t_1 + t_2 + d_2 + t_4 + t_6 + t_8, d_1 + t_2 + t_3 + d_3 + t_5 + t_7 + t_8) = d_1 + t_1 + t_2 + d_2 + t_4 + t_6$$

We have

$$x_1(k) = \max(x_1(k-1) + d_1, u(k) + t_1) \quad (18)$$

$$x_2(k) = \max(x_1(k-1) + 2d_1 + t_2, x_2(k-1) + d_2, u(k) + \delta_1) \quad (19)$$

$$x_3(k) = \max(x_1(k-1) + 2d_1 + t_3, x_3(k-1) + d_3, u(k) + \delta_2) \quad (20)$$

$$x_4(k) = \max(x_1(k-1) + 2d_1 + t_2 + d_2 + t_4, x_2(k-1) + 2d_2 + t_4, u(k) + \delta_3) \quad (21)$$

$$x_5(k) = \max(x_1(k-1) + 2d_1 + t_3 + d_3 + t_5, x_3(k-1) + 2d_3 + t_5, u(k) + \delta_4) \quad (22)$$

$$x_6(k) = \max(x_1(k-1) + 2d_1 + t_2 + d_2 + t_4 + t_6, x_2(k-1) + 2d_2 + t_4 + t_6, x_4(k-1) + d_4, u(k) + \delta_5) \quad (23)$$

$$x_7(k) = \max(x_1(k-1) + 2d_1 + t_3 + d_3 + t_5 + t_7, x_3(k-1) + 2d_3 + t_5 + t_7, x_5(k-1) + d_5, u(k) + \delta_6) \quad (24)$$

$$x_8(k) = \max(x_1(k-1) + 2d_1 + t_8 + \delta_7, x_2(k-1) + 2d_2 + t_4 + t_6 + t_8, x_3(k-1) + 2d_3 + t_5 + t_7 + t_8, x_4(k-1) + d_4 + t_8, x_5(k-1) + d_5 + t_8, x_6(k-1) + d_6, u(k) + \delta_8) \quad (25)$$

$$y(k) = x_n(k) + d_n + t_n \quad (26)$$

Converting equations (17-25) a system of Tropical equations, an equivalent detail matrix equations is obtained. Thus;

$$\begin{pmatrix} d_1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 2d_1t_1 & d_2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 2d_1t_3 & \varepsilon & d_3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 2d_1t_2d_2t_4 & 2d_2t_4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 2d_1t_3d_3t_5 & \varepsilon & 2d_3t_5 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 2d_1t_2d_2t_4t_6 & 2d_2t_4t_6 & \varepsilon & d_4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 2d_1t_3d_3t_5t_7 & \varepsilon & 2d_3t_5t_7 & \varepsilon & d_5 & \varepsilon & \varepsilon & \varepsilon \\ 2d_1t_8\delta_7 & 2d_2t_4t_6t_7 & 2d_3t_5t_7t_8 & d_4t_8 & d_5t_8 & d_6 & \varepsilon & \varepsilon \end{pmatrix} \otimes x(k-1) \oplus \begin{pmatrix} t_1 \\ \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \end{pmatrix} \otimes u(k) \quad (26)$$

$$y(k) = (\varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad d_4 + t_{10})x(k)$$

Where $x(k) = [x_1(k), x_2(k), x_3(k), \dots, x_8(k)]^T$.

3.2 Numerical examples for the model

After introduction of numerical values into the model (26) above as follows:

$d_1 = 7, d_2 = 4, d_3 = 7, d_4 = 10, d_5 = 8, d_6 = 1$ and $t_1 = 2, t_2 = 3, t_3 = 5, t_4 = 4, t_5 = 2, t_6 = 3, t_7 = 2, t_8 = 4, t_9 = 3$, and $t_{10} = 5$

From (26), we obtain

$$\begin{pmatrix} 7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 12 & 4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 14 & \varepsilon & 7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 20 & 10 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 23 & \varepsilon & 11 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 23 & 13 & \varepsilon & 10 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 25 & \varepsilon & 13 & \varepsilon & 8 & \varepsilon & \varepsilon & \varepsilon \\ 29 & 17 & 17 & 14 & 12 & 1 & \varepsilon & \varepsilon \end{pmatrix} \otimes x(k-1) \oplus \begin{pmatrix} 2 \\ 12 \\ 14 \\ 18 \\ 23 \\ 23 \\ 25 \\ 30 \end{pmatrix} \otimes u(k)$$

$$y(k) = (\varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad 6)x(k)$$

Where $x(k) = [x_1(k), x_2(k), x_3(k), \dots, x_8(k)]^T$.

Implies that

$$A = \begin{pmatrix} 7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 12 & 4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 14 & \varepsilon & 7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 20 & 10 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 23 & \varepsilon & 11 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 23 & 13 & \varepsilon & 10 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 25 & \varepsilon & 13 & \varepsilon & 8 & \varepsilon & \varepsilon & \varepsilon \\ 29 & 17 & 17 & 14 & 12 & 1 & \varepsilon & \varepsilon \end{pmatrix}$$

Iterations according equation $x(k)=A \otimes x(k-1)$.

Since $x(0) = [x_1(0), x_2(0), x_3(0), \dots, x_8(0)]^T = [0, 0, 0, \dots, 0]^T$ such that the condition that initially all buffers are empty.

Gives

$$x(1) = A \otimes x(0) = [7 \quad 12 \quad 14 \quad 20 \quad 23 \quad 23 \quad 25 \quad 29]^T$$

$$x(2) = A \otimes x(1) = [14 \quad 19 \quad 21 \quad 27 \quad 30 \quad 30 \quad 32 \quad 36]^T$$

$$x(3) = A \otimes x(2) = [21 \quad 26 \quad 28 \quad 34 \quad 37 \quad 37 \quad 39 \quad 43]^T$$

$$x(4) = A \otimes x(3) = [28 \quad 33 \quad 35 \quad 41 \quad 44 \quad 44 \quad 45 \quad 50]^T$$

$$x(5) = A \otimes x(4) = [35 \quad 40 \quad 42 \quad 48 \quad 51 \quad 51 \quad 53 \quad 58]^T$$

$$x(6) = A \otimes x(5) = [42 \quad 47 \quad 49 \quad 55 \quad 58 \quad 58 \quad 60 \quad 65]^T$$

$$x(7) = A \otimes x(6) = [49 \quad 54 \quad 56 \quad 62 \quad 65 \quad 65 \quad 67 \quad 71]^T$$

$$x(8) = A \otimes x(7) = [56 \quad 61 \quad 63 \quad 69 \quad 72 \quad 72 \quad 73 \quad 76]^T$$

$$x(9) = A \otimes x(8) = [63 \quad 68 \quad 70 \quad 76 \quad 79 \quad 79 \quad 81 \quad 85]^T$$

$$\text{Hence } x(9) - x(1) = \begin{pmatrix} 63 \\ 68 \\ 70 \\ 76 \\ 79 \\ 79 \\ 81 \\ 85 \end{pmatrix} - \begin{pmatrix} 7 \\ 12 \\ 14 \\ 20 \\ 23 \\ 23 \\ 25 \\ 29 \end{pmatrix} = \begin{pmatrix} 56 \\ 56 \\ 56 \\ 56 \\ 56 \\ 56 \\ 56 \\ 56 \end{pmatrix},$$

The eigenvalue $\lambda(x) = \frac{x(9)-x(1)}{9-1} = \frac{56}{8} = 7$, which is the average maximum cycle (period) of production. The Eigen vector can be found by using this form below;

$$V(A) = x(8) \oplus \lambda(x) \otimes x(7) \oplus \lambda(x)^{\otimes 2} \otimes x(6) \oplus \lambda(x)^{\otimes 3} \otimes x(5) \oplus \lambda(x)^{\otimes 4} \otimes x(4) \oplus \lambda(x)^{\otimes 5} \otimes x(3) \oplus \lambda(x)^{\otimes 6} \otimes x(2) \oplus \lambda(x)^{\otimes 7} \otimes x(1)$$

$$\begin{pmatrix} 56 \\ 61 \\ 63 \\ 69 \\ 72 \\ 72 \\ 73 \\ 76 \end{pmatrix} \oplus 7 \otimes \begin{pmatrix} 49 \\ 54 \\ 56 \\ 62 \\ 65 \\ 65 \\ 67 \\ 71 \end{pmatrix} \oplus 7^{\otimes 2} \otimes \begin{pmatrix} 42 \\ 47 \\ 49 \\ 55 \\ 58 \\ 60 \\ 65 \end{pmatrix} \oplus 7^{\otimes 3} \otimes \begin{pmatrix} 35 \\ 40 \\ 42 \\ 48 \\ 51 \\ 53 \\ 58 \end{pmatrix} \oplus 7^{\otimes 4} \otimes \begin{pmatrix} 28 \\ 33 \\ 35 \\ 41 \\ 44 \\ 44 \\ 45 \\ 50 \end{pmatrix} \oplus 7^{\otimes 5} \otimes \begin{pmatrix} 21 \\ 26 \\ 28 \\ 34 \\ 37 \\ 37 \\ 39 \\ 43 \end{pmatrix} \oplus 7^{\otimes 6} \otimes \begin{pmatrix} 14 \\ 19 \\ 21 \\ 27 \\ 30 \\ 30 \\ 32 \\ 36 \end{pmatrix} \oplus 7^{\otimes 7} \otimes \begin{pmatrix} 7 \\ 12 \\ 14 \\ 20 \\ 23 \\ 23 \\ 25 \\ 29 \end{pmatrix}$$

$$\begin{pmatrix} 56 \oplus 56 \oplus 56 \oplus 56 \oplus 56 \oplus 56 \oplus 56 \oplus 56 \\ 61 \oplus 61 \oplus 61 \oplus 61 \oplus 61 \oplus 61 \oplus 61 \oplus 61 \\ 63 \oplus 63 \oplus 63 \oplus 63 \oplus 63 \oplus 63 \oplus 63 \oplus 63 \\ 69 \oplus 69 \oplus 69 \oplus 69 \oplus 69 \oplus 69 \oplus 69 \oplus 69 \\ 72 \oplus 72 \oplus 72 \oplus 72 \oplus 72 \oplus 72 \oplus 72 \oplus 72 \\ 72 \oplus 72 \oplus 72 \oplus 72 \oplus 72 \oplus 72 \oplus 72 \oplus 72 \\ 73 \oplus 74 \oplus 74 \oplus 74 \oplus 73 \oplus 74 \oplus 74 \oplus 74 \\ 76 \oplus 78 \oplus 79 \oplus 79 \oplus 78 \oplus 78 \oplus 78 \oplus 78 \end{pmatrix} = \begin{pmatrix} 56 \\ 61 \\ 63 \\ 69 \\ 72 \\ 72 \\ 74 \\ 79 \end{pmatrix} = x(k)$$

Thus $x(k)$ is Eigen vector of the matrix A subject to verification below

$$A \otimes x(k) = \lambda(x) \otimes x(k) = [63 \ 68 \ 70 \ 76 \ 79 \ 79 \ 81 \ 85]$$

Hence $x(k) = [56 \ 61 \ 63 \ 69 \ 72 \ 72 \ 74 \ 79]$ is the Eigen vector of A that stand as the starting time of each processing stage for the production process schedule. The initial time the system starts working is selected with an eigenvector that has a minimum non-negative component so that the finished process and available materials can be processed immediately. An eigenvector with a minimum non-negative component is obtained when the smallest Eigen vector component is converted to zero. The eigenvector v' which has the minimum non negative component is obtained by taking $\alpha = -56$ so that v' is obtained is shown below.

$$v'(k) = [0 \ 5 \ 7 \ 13 \ 16 \ 16 \ 18 \ 23]$$

So for each processing unit will work with an average period of $\lambda(x) = 7$ hours and the processing task $P_1, 2, P_3, P_4, P_5, S_1, S_2, 6$ successively work for the first process at hours 0,5,7,13,16,16,18,23.

Tasks	Batch	
	1 st	2 nd
P_1	0	24
P_2	5	29
P_3	7	36
P_4	13	49
P_5	16	65
S_1	16	81
S_2	18	99
P_6	23	112

From Table 1 above, it can be converted into working hours to be more representative in accordance with the actual situation on the ground. Based on the normal circumstance company operation starts at 06:00am so that the 0-th hour in Table 1 above can be represented that the first task (P_1) starts working by 06:00am and so on for the other task. The production schedule after referring to working hours is shown in Table 2 below.

Based on Table 2, it is known that in two weeks working day two production processes can be carried out. The first batch starts at 06:00am and then the second batch production starts at 09:00pm. Using the production schedule in Table 2 helps the

manager to know when a good start time to start each task and the production process can run periodically.

Table 2: Production schedule

Task	Task Starting Time	
	1 st batch	2 nd batch
P_1	06:00am	09:00pm
P_2	11:00am	02:00 am
P_3	6:00pm	09:00am
P_4	07:00am	10:00pm
P_5	10:00pm	02:00pm
S_1	02:00am	06:00am
S_2	10:00pm	12:00am
P_6	9:00pm	11:00am

4. Discussion of result

This study provide a scheduling model for manufacturing system and the result found to be the Eigen value of the obtained matrix for the model as $\lambda(x) = 7 \text{ hour}$ stand for the average times production cycles, so for each processing units $P_1, P_2, P_3 \dots P_6$ Successively work for the first process at

$$[0 \ 5 \ 7 \ 13 \ 18 \ 23]^T$$

And stored at 16 (hours) and the Eigen vector is

$$[56 \ 61 \ 63 \ 69 \ 72 \ 72 \ 74 \ 79]^T$$

Which stand for the time instant where the respective production schedules are to start taking place. Based on Table 2, it is known that in two weeks working day two production processes can be carried out. The first batch starts at 06:00am and then the second batch production starts at 09:00pm.

5. Conclusions

This research comes up with production schedule model. The production schedule makes the tasks more effective and efficient because in the process, the idle machine will immediately start the next process. We use matrix operations, solve systems of tropical equations, and have existence and meaning for eigenvalues and eigenvectors. It described how a tropical model for a manufacturing system can be constructed, and an optimal schedule was found without optimization. The scheduling of

production systems consisting of many stages and different units is considered, where some of the units working simultaneously.

Through applications, it can be have seen that tropical algebra provide interesting tools that can be used to models, formulate and solve many problems of optimization. There are numerous applications of tropical algebra, and they are certainly not limited to only those of manufacturing system. We have tried to present the models in an understandable and reader-friendly way and encourage the reader to seek out new applications of the max-plus algebra.

In terms of management it also makes it easy for manager to know when a good start time to start each task and the production process can run periodically. However, this research is still limited to the assumption that the time when raw materials enter the system together with the time when products leave the system.

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